NWERC 2015 Presentation of solutions

The Jury

2015-11-29

NWERC 2015 solutions

NWERC 2015 Jury

- Per Austrin (KTH Royal Institute of Technology)
- Gregor Behnke (Ulm University)
- Jeroen Bransen (Utrecht University)
- Egor Dranischnikow (CST AG, Darmstadt)
- Tommy Färnqvist (Linköping University)
- Florin Ghesu (Siemens Healthcare/FAU Erlangen-Nürnberg)
- Robin Lee (Google)
- Lukáš Poláček (Spotify)
- Stefan Toman (TU München)
- Tobias Werth (FAU Erlangen-Nürnberg)
- Paul Wild (FAU Erlangen-Nürnberg)

Big thanks to our test solvers

- Michal Forišek (Comenius University)
- Jan Kuipers (AppTornado)

Problem

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Statistics: 131 submissions, 92 accepted

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Possible pitfalls

Reading in the quadkey as integer

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Possible pitfalls

- Result does not fit in an int
- Greedy or brute force approach will not work

Statistics: 194 submissions, 45 accepted

Problem Author: Jeroen Bransen NWERC 2015 solutions

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- r time needed to compile+run the program
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Solution (greedy part)

- best strategy for inserting k printf statements in n LoC equidistant, every $\left\lceil \frac{n}{k+1} \right\rceil$ lines - minimizing the number of LoCs in the next step.
- for *n* LoC the best strategy could be to insert 1, 2, ... or n-1 printf statements.

D – Debugging

Solution (recursive part)

Boils down to calculating the recursion:

$$T(1) = 0$$

$$T(n) = \min_{1 \le k < n} \left\{ k \cdot p + T\left(\left\lceil \frac{n}{k+1} \right\rceil \right) \right\} + r$$

- naive recursion way too slow
- ② naive DP calculates too many not needed states (e.g. n-1, n-2, ...) → $O(n^2)$ too slow
- memoization calculates only the needed states $\rightarrow O(n \log n)$ good enough
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C - Cleaning Pipes

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G – Guessing Camels

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- Counting the number of inversions can be done in $O(n \log n)$.

Statistics: 97 submissions, 13 accepted

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Statistics: 14 submissions, 3 accepted

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- **③** Observation: good workers do **not** affect the current partition

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Statistics: 8 submissions, ?? accepted

Problem

Given some spherical arcs and spherical polygons, how much of the arcs is contained in the polygons?



Solution

Problem Author: Per Austrin NWERC 2015 solutions

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- For each route segment, find all points at which we switch between land and water.
 - We were promised that no tricky cases occur ⇒ only need to find all intersections of route segment with polygon segments.

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 - We were promised that no tricky cases occur ⇒ only need to find all intersections of route segment with polygon segments.
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- Alternate between land and water (we know that we start on land)

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- The great circle on which the arc from P_1 to P_2 runs are the unit vectors orthogonal to $P_1 \times P_2$
- ② Intersection of $P_1 \frown P_2$ and $Q_1 \frown Q_2$ (if it exists) must lie on both great circles ⇔ orthogonal to both $P_1 \times P_2$ and $Q_1 \times Q_2$.
Flight Plan Evaluation

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- Only two possible candidate intersection points: $\pm \frac{(P_1 \times P_2) \times (Q_1 \times Q_2)}{\|(P_1 \times P_2) \times (Q_1 \times Q_2)\|_2}$

Watch out for division by zero (cocircular arcs).

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To test if candidate point *R* is on *P*₁ → *P*₂: check if d(*P*₁, *R*) + d(*R*, *P*₂) = d(*P*₁, *P*₂).

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- Only two possible candidate intersection points: $\begin{array}{c} (P_1 \times P_2) \times (Q_1 \times Q_2) \end{array}$

$$\frac{1}{\|(P_1 \times P_2) \times (Q_1 \times Q_2)\|_2}$$

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Statistics: 9 submissions, 2 accepted

Random numbers produced by the jury

- 1217 number of posts made in the jury's forum. (NWERC 2014: 1087)
 - 915 commits made to the problem set repository. (NWERC 2014: 671)
 - 416 number of lines of code used in total by the shortest judge solutions to solve the entire problem set. (NWERC 2014: 380)
- 16.6 average number of jury solutions per problem, including incorrect ones. (NWERC 2014: 16.7)
- 62.5% fraction of judge solutions to problem B that were successfully challenged by a single test case 4 days before the contest.