0x13 Ural Championship 2015

Problem analysis

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A. The first day of school

Problem statement

- We have 3 days of school
- Each day has 4 time slots
- Each time slot can contain 1 lesson
- Each lesson has a name
- Name contains of up to 5 words, each up to 10 letters
- We need to print a timetable as an actual table

A. The first day of school

Problem solution

- Each cell has same width (10 characters)
- All cells in a row has equal height, which is equal to maximum height of cell in this row
- Height for single cell is calculated from the name of lesson in this cell
- The only thing we need is correct understanding of statement and thorough implementation

B. Maths

Problem statement

- We need to find a sequence a_i of length n
- Constraint: $\forall k \geq 2 : cnt(\sum_{i=1}^{k} a_i) = a_k$, where $cnt(x)$ is equal to amount of different positive divisors of x

B. Maths

Problem solution

- Dynamic programming
- dp_i is the maximum length of such sequence that can be finished with number i
- $dp_i = dp_{i-cnt(i)} + 1$
- Answer is i for which $dp_i = n$
- There is always an answer in $[1; 2 \times 10^6]$ (proved by experiment)

- We have a time range $[l; r]$, l and r are year numbers
- We need to calculate amount of years with all possible counts of Friday, 13th
- $l, r \leq 10^9$

C. History

Leap years rules

- Year is not leap, if it's number isn't divided by 4
- Year is not leap, if it's number is divided by 100, but isn't divided by 400
- All the other years are leap
- So the years x and $x + 400$ are either both leap or both not leap

C. History

Finding a cycle

- Every 400 years contain the same amount of days in it (let's call it X)
- Every Every 2800 years contain the same amount of days $7 \times X$, and this amount is divided by 7
- So the years x and $x + 2800$ always start from the same day of week

C. History

Solution

- We can calculate answer for the first 2800 years with day-by-day modeling
- Then we can multiply the answer on $\frac{y-x+1}{2800}$
- And then we can model remaining years (there are less then 2800 of them)

D. Chemistry

Problem statement

- We have n tubes with 1 unit of liquid in each
- From two tubes with x and y ($x \geq y$) units we can get two tubes with $x - y$ and $2 \times y$ units
- We need to get a tube with k units

D. Chemistry
Theorem

- For odd n we can get any $x \in [0; n-1]$
- For even n we can get any $x \in [0; n-2]$
- Algorithm will be derived from proof
- We'll use induction
- Base $(n = 3,\ n = 4)$ is obvious

D. Chemistry

Induction step for even n

- Consider even n
- For $n - 1$ we can get $x \in [0; (n-1) - 1 = n - 2]$

D. Chemistry

Observation

- Consider two tubes with a and b, $a > b$
- We can get two tubes with $2 \times b = (2 \times b) \mod n$ and $a - b = a - n + a = 2 \times a - n = (2 \times a) \mod n$
- So for one pour action we double amount in every tube by modulo 2
- So for t pour actions with a and b we get $(2^t \times a) \mod n$ and $(2^t \times b) \mod n$

D. Chemistry

Induction step for odd n

- Consider odd n
- For $n-2$ we can get $x \in [0; (n-2)-1 = n-3]$
- Let's get two tubes with $n-4$ and 4 units
- Let's do $\phi(n)-2$ pour actions
- We'll get $2^{\phi(n)} \mod n = 1$ unit in second tube
- And we'll get $n-1$ unit in the first tube
- With one more pour action we'll get $n-2$ in the first tube

D. Chemistry

The rest of the proof (1)

- Theorem is almost proved
- Odd n can't be got (it should be the last pour action, but it's odd)
- For even n $n-1$ can't be got (it should be got by pouring something from n, but in that case all other units are empty)
- For even n n can't be got (proved on the next slide)

D. Chemistry

The rest of the proof (2)

- Consider going backwards
- We have one tube with n units and $n - 1$ empty ones
- We can split any tube with even amount of units onto two parts
- If n is not some degree of 2, it will always be divided by some odd $k \geq 3$
- Degree of 2 obviously can be got

E. 3d-modeling

Problem statement

- We have 2 straight lines
- We need to rotate one of them around some axis so that they became equal

- Let's set rotating angle to 180°
- This way rotation is just reflecting from axis

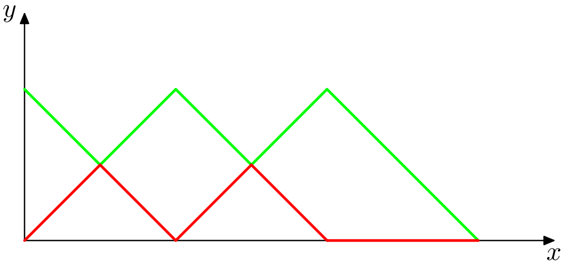
E. 3d-modeling

Solution

- Let's say that $\vec{a}$ and $\vec{b}$ are direction vectors for our lines, and $\vec{c}$ is direction vector for axis
- Let's say that X and Y are some pair of closest points on our lines
- If axis will go through $\frac{X+Y}{2}$, then Y will be reflected into X
- If we set $\vec{c} = \vec{a} + \vec{b}$, then second line will be reflected into first line

F. Physics
Problem statement (1)

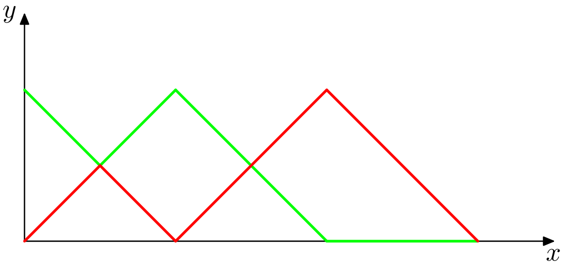
- We have two polygonal chains with some common points



F. Physics

Problem statement (2)

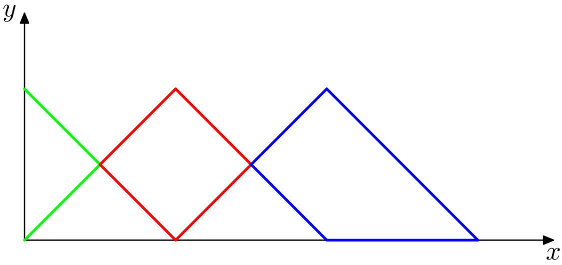
- We need to reassign some segments so that it would still be two polygonal chains
- Area under first chain should be equal to area under second



F. Physics

Solution idea (1)

- We have only 30 common points
- We can reassign only whole set of segments between two consequent common points
- Let's use common points to split our chains onto parts



- For every part we'll calculate areas under it's top border and bottom border
- We have two sequences a_n and b_n
- We need to find such set of indices S that $\sum_{i \in S} a_i + \sum_{i \notin S} b_i = \sum_{i \notin S} a_i + \sum_{i \in S} b_i$

- Let's split all indices onto two halves
- For every part we'll calculate all possible differences between $\sum a$ and $\sum b$
- For every possible difference in first half we'll try to find a matching difference in second half
- After that we'll only need to eliminate consequent triplets of points on same line

G. Physical education

Problem statement

- We sort all numbers from 1 to n ($n \leq 10^9$) by sum of digits and then by value
- Need to find amount of numbers that didn't change their positions

G. Physical education

Problem solution

- Let's consider all numbers with equal digit sum
- They are sorted by their values
- We can precalculate set of positions they will take
- Difference between two consequent number is always greater, then 1
- So there is at most 1 number which hasn't change it's position
- It can be found by binary search
- Query *k-th number less then n with sum of digits equal to x* can be supported with dynamic programming

- Construct a planar graph with 16 vertices and ≤ 300000 simple cycles.

H. Biology

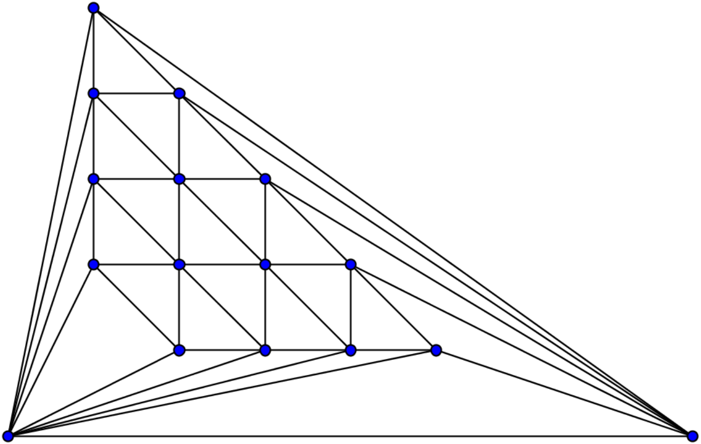
Solution

- As for most constructive problems, write a local testing utility. Don't attempt to submit blindly! For this problem it's standard DP algorithm working in $O(2^n n^2)$ time.
- After coding a local checker, try different approachs by hand, if they work, it's great!
- If that didn't help, try generating random tests. For this problem, generate random points and add random edges until you can't add any. It's also a good idea to randomly flip edges while the answer still increases.

H. Biology

Solution

- A picture of a solution, courtesy of nk.karpov @ Codeforces



J. Urban geography

Problem statement

- Given a weighted undirected graph
- Find the connected subgraph with minimum difference between maximum and minimum edge

J. Urban geography

Solution

- Sort all edges by weight, then run a binary search on problem's answer.
- How to check if the answer is $\leq M$? Use two pointers technique to add and remove edges and check if graph becomes connected.
- It's hard to support all the above operations online, let's do this offline! Dynamic connectivity offline algorithm can be implemented to work in $O(M \log^2 N)$.
- Summary: binary search + offline dynamic connectivity gives $O(M \log^2 N \log C)$ algorithm.